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Universality of thermal hadron production in pp, $p\bar{p}$ and e^+e^- collisions

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Abstract

It is shown that the hadron production in high energy pp, $p\bar{p}$ and e^+e^- collisions, calculated by assuming that particles originate in hadron gas fireballs at thermal and partial chemical equilibrium, agrees very well with the data. The temperature of the hadron gas fireballs, determined by fitting hadron abundances, does not seem to depend on the centre of mass energy and kind of reaction, having a nearly constant value of about 170 MeV. This finding supports a universal hadronization mechanism in all kinds of reactions consisting in a parton-hadron transition at critical values of temperature and pressure.

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1 Introduction

The thermodynamical approach to hadron production in hadronic collisions was originally introduced by Hagedorn [1] almost thirty years ago. One of the most important phenomenological indication of a thermal-like multihadron production in those reactions at high energy was found in the universal slope of transverse mass (i.e. $m_T = \sqrt{p_T^2 + m^2}$) spectra [2] which is nowadays still being used extensively in heavy ions reactions, although it has been realized that transverse collective motions of the hadron gas may significantly distort the primitive thermal m_T -spectrum [3]. A much better probe of the existence of locally thermalized sources in hadronic collisions is the overall production rate of single hadron species which, being a Lorentz-invariant quantity, is not affected by local collective motions of the hadron gas. The analysis of hadron production rates with the thermodynamical *ansatz* implies that inter-species chemical equilibrium is attained, which is a tighter requirement with respect to a thermal-kinetical intra-species equilibrium assumed in the analysis of m_T spectra. However, since hadron abundances can provide a cleaner indication of chemical and thermal equilibrium with respect to transverse mass spectra, we have focused our attention on the former issue.

The smallness of the collision systems studied here requires appropriate theoretical tools: in order to properly compare theoretical predicted multiplicities to experimental ones, the use of statistical mechanics in its canonical form is mandatory, that means exact quantum numbers conservation is required, unlike in the grand-canonical formalism [4]. It will be shown indeed that particle average particle multiplicities in small systems are heavily affected by conservation laws well beyond what the use of chemical potentials predicts.

The thermodynamical approach to hadron production in e^+e^- collisions [5] is here extended to pp and $p\bar{p}$ collisions; some assumptions which were made for e^+e^- , in particular the restriction to two-jet events, are released; all calculations are performed with a bigger symmetry group (actually by also taking into account the conservation of electric charge). With this generalized model, we also update our results for e^+e^- collisions.

2 The model

In ref. [5, 6] a thermodynamical model of hadron production in e^+e^- collisions was developed on the basis of the following assumption: the hadronic jets observed in the final state of a $e^+e^- \rightarrow q\bar{q}$ event must be identified with hadron gas phases having a collective motion. This identification is valid at the decoupling time, when hadrons stop interacting after their formation and (possibly) a short expansion (*freeze-out*). Throughout this paper we will refer to such hadron gas phases with a collective motion as *fireballs*, following refs. [1, 2]. Since most events in a $e^+e^- \rightarrow q\bar{q}$ reaction are two-jet events, it was assumed that two fireballs are formed and that their internal properties, namely quantum numbers, are related to those of the corresponding primary quarks. In the so-called *correlated jet scheme* correlations between the quantum numbers of the two fireballs were allowed beyond the simple correspondance between the fireball and the parent quark quantum numbers. This scheme turned out to be in better agreement with the data than a correlation-free scheme [5].

The more complicated structure of a hadronic collision does not allow a straightforward extension of this model. If the assumption of hadron gas fireballs is maintained, the possibility of an arbitrary number of fireballs with an arbitrary configuration of quantum numbers should be taken into account. To be specific, let us define a vector $\mathbf{Q} = (Q, N, S, C, B)$ with integer components equal to the electric charge, baryon number, strangeness, charm and beauty respec-

tively. We assume that the final state of a pp or a $p\bar{p}$ interaction consists of a set of N fireballs, each with its own four-vector $\beta_i = u_i/T_i$, where T_i is the temperature and $u_i = (\gamma_i, \beta_i \gamma_i)$ is the four-velocity [7], quantum numbers \mathbf{Q}_i^0 and volume in the rest frame V_i . The quantum vectors \mathbf{Q}_i^0 must fulfill the overall conservation constraint $\sum_{i=1}^N \mathbf{Q}_i^0 = \mathbf{Q}^0$ where \mathbf{Q}^0 is the vector of the initial quantum numbers, that is $\mathbf{Q}^0 = (2, 2, 0, 0, 0)$ in a pp collision and $\mathbf{Q}^0 = (0, 0, 0, 0, 0)$ in a $p\bar{p}$ collision.

The invariant partition function of a single fireball is, by definition:

$$Z_i(\mathbf{Q}_i^0) = \sum_{\text{states}} e^{-\beta_i \cdot P_i} \delta_{\mathbf{Q}_i, \mathbf{Q}_i^0}, \quad (1)$$

where P_i is its total four-momentum. The factor $\delta_{\mathbf{Q}_i, \mathbf{Q}_i^0}$ is the usual Kronecker tensor, which forces the sum to be performed only over the fireball states whose quantum numbers \mathbf{Q}_i are equal to the particular set \mathbf{Q}_i^0 . It is worth emphasizing that this partition function corresponds to the *canonical* ensemble of statistical mechanics since only the states fulfilling a fixed chemical requirement, as expressed by the factor $\delta_{\mathbf{Q}_i, \mathbf{Q}_i^0}$, are involved in the sum (1).

By using the integral representation of $\delta_{\mathbf{Q}_i, \mathbf{Q}_i^0}$:

$$\delta_{\mathbf{Q}_i, \mathbf{Q}_i^0} = \frac{1}{(2\pi)^5} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} d^5\phi \, e^{i(\mathbf{Q}_i^0 - \mathbf{Q}_i) \cdot \phi}, \quad (2)$$

Eq. (1) becomes:

$$Z_i(\mathbf{Q}_i^0) = \sum_{\text{states}} \frac{1}{(2\pi)^5} \int_0^{2\pi} \dots \int_0^{2\pi} d^5\phi \, e^{-\beta_i \cdot P_i} e^{i(\mathbf{Q}_i^0 - \mathbf{Q}_i) \cdot \phi}. \quad (3)$$

This equation could also have been derived from the general expression of partition function of systems with internal symmetry [8, 9] by requiring a $U(1)^5$ symmetry group, each $U(1)$ corresponding to a conserved quantum number; that was the procedure taken in ref. [5].

The sum over states in Eq. 3 can be worked out quite straightforwardly for a hadron gas of N_B boson species and N_F fermion species. A state is specified by a set of occupation numbers $\{n_{j,k}\}$ for each phase space cell k and for each particle species j . Since $P_i = \sum_{j,k} p_k n_{j,k}$ and $\mathbf{Q}_i = \sum_{j,k} \mathbf{q}_j n_{j,k}$, where $\mathbf{q}_j = (Q_j, N_j, S_j, C_j, B_j)$ is the quantum number vector associated to the j^{th} particle species, the partition function (3) reads, after summing over states:

$$Z_i(\mathbf{Q}_i^0) = \frac{1}{(2\pi)^5} \int d^5\phi \, e^{i\mathbf{Q}_i^0 \cdot \phi} \exp \left[\sum_{j=1}^{N_B} \sum_k \log(1 - e^{-\beta_i \cdot p_k - i\mathbf{q}_j \cdot \phi})^{-1} + \sum_{j=1}^{N_F} \sum_k \log(1 + e^{-\beta_i \cdot p_k - i\mathbf{q}_j \cdot \phi}) \right]. \quad (4)$$

The last expression of the partition function is manifestly Lorentz-invariant because the sum over phase space is a Lorentz-invariant operation which can be performed in any frame. The most suitable one is the fireball rest frame, where the four-vector β_i reduces to:

$$\beta_i = \left(\frac{1}{T_i}, 0, 0, 0 \right) \quad (5)$$

T_i being the temperature of the fireball. Moreover, the sum over phase space cells in Eq. (4) can be turned into an integration over momentum space going to the continuum limit:

$$\sum_k \longrightarrow (2J_j + 1) \frac{V}{(2\pi)^3} \int d^3p, \quad (6)$$

where V is the fireball volume and J_j the spin of the j^{th} hadron. As in previous studies on e^+e^- collisions [5] and heavy ions collisions [10], we supplement the ordinary statistical mechanics formalism with a strangeness suppression factor γ_s accounting for a partial strangeness phase space saturation; actually the Boltzmann factor $e^{-\beta \cdot p_k}$ of any hadron species containing s strange valence quarks or antiquarks is multiplied by γ_s^s . With the transformation (6) and choosing the fireball rest frame to perform the integration, the sum over phase space in Eq. (4) becomes:

$$\sum_k \log(1 \pm e^{-\beta_i \cdot p_k - i\mathbf{q}_j \cdot \phi})^{\pm 1} \longrightarrow \frac{(2J_j + 1) V_i}{(2\pi)^3} \int d^3p \log(1 \pm \gamma_s^{s_j} e^{-\sqrt{p^2 + m_j^2}/T_i - i\mathbf{q}_j \cdot \phi})^{\pm 1} \equiv V_i F_j(T_i, \gamma_s) \quad (7)$$

where the upper sign is for fermions, the lower for bosons and V_i is the fireball volume in its rest frame; the function $F_j(T_i, \gamma_s)$ is a shorthand notation of the momentum integral in Eq. (7). Hence, the partition function (4) can be written:

$$Z_i(\mathbf{Q}_i^0) = \frac{1}{(2\pi)^5} \int d^5\phi e^{i\mathbf{Q}_i^0 \cdot \phi} \exp[V_i \sum_j F_j(T_i, \gamma_s)] . \quad (8)$$

The mean number $\langle n_j \rangle_i$ of the j^{th} particle species in the i^{th} fireball can be derived from $Z(\mathbf{Q}_i^0)$ by multiplying the Boltzmann factor $\exp(-\sqrt{p^2 + m_j^2}/T)$, in the function F_j in Eq. (8) by a fictitious fugacity λ_j and taking the derivative of $\log Z_i(\mathbf{Q}_i^0, \lambda_j)$ with respect to λ_j at $\lambda_j = 1$:

$$\langle n_j \rangle_i = \frac{\partial}{\partial \lambda_j} \log Z_i(\mathbf{Q}_i^0, \lambda_j) \Big|_{\lambda_j=1} . \quad (9)$$

The partition function $Z_i(\mathbf{Q}_i, \lambda_j)$ supplemented with the λ_j factor is still a Lorentz-invariant quantity and so is the mean number $\langle n_j \rangle_i$. From a more physical point of view, this means that the average multiplicity of any hadron does not depend on fireball collective motion, unlike its mean number in a particular momentum state.

The overall average multiplicity of the j^{th} hadron, for a set of N fireballs in a certain quantum configuration $\{\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0\}$ is the sum of all mean numbers of that hadron in each fireball:

$$\langle n_j \rangle = \sum_{i=1}^N \frac{\partial}{\partial \lambda_j} \log Z_i(\mathbf{Q}_i^0, \lambda_j) \Big|_{\lambda_j=1} = \frac{\partial}{\partial \lambda_j} \log \prod_{i=1}^N Z_i(\mathbf{Q}_i^0, \lambda_j) \Big|_{\lambda_j=1} . \quad (10)$$

In general, as the quantum number configurations may fluctuate, hadron production should be further averaged over all possible fireballs configurations $\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0$ fulfilling the constraint $\sum_{i=1}^N \mathbf{Q}_i^0 = \mathbf{Q}^0$. To this end, suitable weights $w(\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0)$, representing the probability of configuration $\{\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0\}$ to occur for a set of N fireballs, must be introduced. Basic features of those weights are:

$$\begin{aligned} w(\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0) &= 0 \quad \text{if} \quad \sum_{i=1}^N \mathbf{Q}_i^0 \neq \mathbf{Q}^0, \\ \sum_{\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0} w(\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0) &= 1. \end{aligned} \quad (11)$$

For the overall average multiplicity of hadron j we get:

$$\langle\langle n_j \rangle\rangle = \sum_{\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0} w(\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0) \frac{\partial}{\partial \lambda_j} \log \prod_{i=1}^N Z_i(\mathbf{Q}_i^0, \lambda_j) \Big|_{\lambda_j=1} . \quad (12)$$

There are infinitely many possible choices of the weights $w(\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0)$, all of them equally legitimate. However, one of them is the most pertinent from the statistical mechanics point of view, namely:

$$w(\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0) = \frac{\delta_{\Sigma_i \mathbf{Q}_i^0, \mathbf{Q}^0} \prod_{i=1}^N Z_i(\mathbf{Q}_i^0)}{\sum_{\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0} \delta_{\Sigma_i \mathbf{Q}_i^0, \mathbf{Q}^0} \prod_{i=1}^N Z_i(\mathbf{Q}_i^0)} . \quad (13)$$

It can be shown indeed that this choice corresponds to the minimal deviation from statistical equilibrium of the system as a whole. In fact, putting weights (13) in the Eq. (12), one obtains:

$$\langle\langle n_j \rangle\rangle = \frac{\partial}{\partial \lambda_j} \log \sum_{\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0} \delta_{\Sigma_i \mathbf{Q}_i^0, \mathbf{Q}^0} \prod_{i=1}^N Z_i(\mathbf{Q}_i^0, \lambda_j) \Big|_{\lambda_j=1} . \quad (14)$$

This means that the average multiplicity of any hadron can be derived from the following function of \mathbf{Q}^0 :

$$Z(\mathbf{Q}^0) = \sum_{\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0} \delta_{\Sigma_i \mathbf{Q}_i^0, \mathbf{Q}^0} \prod_{i=1}^N Z_i(\mathbf{Q}_i^0) , \quad (15)$$

with the same recipe given for a single fireball in Eq. (9). By using expression (1) for the partition functions $Z_i(\mathbf{Q}_i^0)$, Eq. (15) becomes:

$$Z(\mathbf{Q}^0) = \sum_{\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0} \delta_{\Sigma_i \mathbf{Q}_i^0, \mathbf{Q}^0} \prod_{i=1}^N \sum_{\text{states}_i} e^{-\beta_i \cdot P_i} \delta_{\mathbf{Q}_i^0, \mathbf{Q}_i} . \quad (16)$$

Since

$$\sum_{\mathbf{Q}_1^0, \dots, \mathbf{Q}_N^0} \delta_{\Sigma_i \mathbf{Q}_i^0, \mathbf{Q}^0} \delta_{\mathbf{Q}_i^0, \mathbf{Q}_i} = \delta_{\Sigma_i \mathbf{Q}_i, \mathbf{Q}^0} , \quad (17)$$

the function (16) can be written as

$$Z(\mathbf{Q}^0) = \sum_{\text{states}_1} \dots \sum_{\text{states}_N} e^{-\beta_1 \cdot P_1} \dots e^{-\beta_N \cdot P_N} \delta_{\Sigma_i \mathbf{Q}_i, \mathbf{Q}^0} . \quad (18)$$

This expression demonstrates that $Z(\mathbf{Q}^0)$ may be properly called the *global partition function* of a system split into N subsystems which are in mutual chemical equilibrium but not in mutual thermal and mechanical equilibrium. Indeed it is a Lorentz-invariant quantity and, in case of complete equilibrium, i.e. $\beta_1 = \beta_2 = \dots = \beta_N \equiv \beta$, it would reduce to:

$$Z(\mathbf{Q}^0) = \sum_{\text{states}_1} \dots \sum_{\text{states}_N} e^{-\beta \cdot (P_1 + \dots + P_N)} \delta_{\Sigma_i \mathbf{Q}_i, \mathbf{Q}^0} = \sum_{\text{states}} e^{-\beta \cdot P} \delta_{\mathbf{Q}, \mathbf{Q}^0} , \quad (19)$$

which is the basic definition of the partition function.

To summarize, the choice of weights (13) allows the construction of a system which is out of equilibrium only by virtue of its subdivision into several parts having different temperatures and velocities. Another very important consequence of that choice is the following: if we assume that the freeze-out temperature of the various fireballs is constant, that is $T_1 = \dots = T_N \equiv T$,

and that the strangeness suppression factor γ_s is constant too, then the global partition function (18) has the following expression:

$$Z(\mathbf{Q}^0) = \frac{1}{(2\pi)^5} \int d^5\phi \, e^{i\mathbf{Q}^0 \cdot \phi} \exp[(\sum_i V_i) \sum_j F_j(T, \gamma_s)] . \quad (20)$$

Here the V_i 's are the fireball volumes in their own rest frames; a proof of (20) is given in ref. [6]. Eq. (20) demonstrates that the global partition function has the same functional form (3), (4), (8) as the partition function of a single fireball, once the volume V_i is replaced by the *global volume* $V \equiv \sum_{i=1}^N V_i$. Note that the global volume absorbs any dependence of the global partition function (20) on the number of fireballs N . Thus, possible variations of the number N and the size V_i of fireballs on an event by event basis can be turned into fluctuations of the global volume. In the remainder of this section these fluctuations will be ignored; in Sect. 3 it will be shown that they do not affect any of the following results on the average hadron multiplicities.

The average multiplicity of the j^{th} hadron can be determined with the formulae (14)-(15), by using expression (20) for the function $Z(\mathbf{Q}^0)$:

$$\begin{aligned} \langle\langle n_j \rangle\rangle &= \frac{1}{(2\pi)^5} \int d^5\phi \, e^{i\mathbf{Q}^0 \cdot \phi} \exp[V \sum_j F_j(T, \gamma_s)] \\ &\times \frac{(2J_j + 1) V}{(2\pi)^3} \int \frac{d^3p}{\gamma_s^{-s_j} \exp(\sqrt{p^2 + m_j^2}/T + i\mathbf{q}_j \cdot \phi) \pm 1} , \end{aligned} \quad (21)$$

where the upper sign is for fermions and the lower for bosons. This formula can be written in a more compact form as a series:

$$\langle\langle n_j \rangle\rangle = \sum_{n=1}^{\infty} (\mp 1)^{n+1} \gamma_s^{ns_j} z_{j(n)} \frac{Z(\mathbf{Q}^0 - n\mathbf{q}_j)}{Z(\mathbf{Q}^0)} , \quad (22)$$

where the functions $z_{j(n)}$ are defined as:

$$z_{j(n)} \equiv (2J_j + 1) \frac{V}{(2\pi)^3} \int d^3p \exp(-n\sqrt{p^2 + m_j^2}/T) = (2J_j + 1) \frac{VT}{2\pi^2 n} m_j^2 K_2\left(\frac{nm_j}{T}\right) . \quad (23)$$

K_2 is the McDonald function of order 2. Eq. (22) is the final expression for the average multiplicity of hadrons at freeze-out. Accordingly, the production rate of a hadron species depends only on its spin, mass, quantum numbers and strange quark content. The *chemical factors* $Z(\mathbf{Q}^0 - n\mathbf{q}_j)/Z(\mathbf{Q}^0)$ in Eq. (22) are a typical feature of the canonical approach due to the requirement of exact conservation of the initial set of quantum numbers. These factors suppress or enhance production of particles according to the vicinity of their quantum numbers to the initial \mathbf{Q}^0 vector. The behaviour of $Z(\mathbf{Q})$ as a function of electric charge, baryon number and strangeness for suitable T , V and γ_s values is shown in Fig. 1; for instance, it is evident that the baryon chemical factors $Z(0, N, 0, 0, 0)/Z(0, 0, 0, 0, 0)$ connected with an initially neutral system play a major role in determining the baryon multiplicities. The ultimate physical reason of “charged” particle ($\mathbf{q}_j \neq 0$) suppression with respect to “neutral” ones ($\mathbf{q}_j = 0$), in a completely neutral system ($\mathbf{Q}^0 = 0$), is the necessity, once a “charged” particle is created, of a simultaneous creation of an anti-charged particle in order to fulfill the conservation laws. In a *finite system* this pair creation mechanism is the more unlikely the more massive is the

lightest particle needed to compensate the first particle's quantum numbers. For instance, once a baryon is created, at least one anti-nucleon must be generated, which is rather unlikely since its mass is much greater than the temperature and the total energy is finite. On the other hand, if a non-strange charged meson is generated, just a pion is needed to balance the total electric charge; its creation is clearly a less unlikely event with respect to the creation of a baryon as the energy to be spent is lower. This argument illustrates why the dependence of $Z(\mathbf{Q})$ on the electric charge is much milder than on baryon number and strangeness (see Fig. 1). In view of that, the dependence of $Z(\mathbf{Q})$ on electric charge was neglected in the previous study on hadron production in e^+e^- collisions [5]. These chemical suppression effects are not accountable in a grand-canonical framework; in fact, in a completely neutral system, all chemical potentials should be set to zero and consequently “charged” particles do not undergo any suppression with respect to “neutral” ones.

3 Other constraints and approximations

Actually, the global partition function (18) has to be modified in $p\bar{p}$ collisions owing to a major effect in such reactions, the *leading baryon effect* [11]. Indeed, the sum (18) includes states with vanishing net absolute value of baryon number, whereas in $p\bar{p}$ collisions at least one baryon-antibaryon pair is always observed. Hence, the simplest way to account for the leading baryon effect is to exclude those states from the sum. Thus, if $|N| = \sum_i |N_i|$ denotes the absolute value of the baryon number of the system, the global partition function (18) should be turned into:

$$Z = \sum_{\text{states}_1} \dots \sum_{\text{states}_N} e^{-\beta_1 \cdot P_1} \dots e^{-\beta_N \cdot P_N} \delta_{\Sigma_i \mathbf{Q}_i, \mathbf{Q}^0} - \sum_{\text{states}_1} \dots \sum_{\text{states}_N} e^{-\beta_1 \cdot P_1} \dots e^{-\beta_N \cdot P_N} \delta_{\Sigma_i \mathbf{Q}_i, \mathbf{Q}^0} \delta_{|N|,0}. \quad (24)$$

The first term, that we define as $Z_1(\mathbf{Q}^0)$, is equal to the function $Z(\mathbf{Q}^0)$ in Eqs. (18), (20), while the second term is the sum over all states having vanishing net absolute value of baryon number. The absolute value of baryon number can be treated as a new independent quantum number so that the processing of the partition function described in Eqs. (1)-(3) can be repeated for the second term in Eq. (24) with a $U(1)^6$ symmetry group. Accordingly, this term can be naturally denoted by $Z_2(\mathbf{Q}^0, 0)$, so that Eq. (24) reads:

$$Z = Z_1(\mathbf{Q}^0) - Z_2(\mathbf{Q}^0, 0). \quad (25)$$

By using the integral representation of $\delta_{|N|,0}$

$$\delta_{|N|,0} = \frac{1}{2\pi} \int_0^{2\pi} d\psi e^{i|N|\psi} \quad (26)$$

in the second term of Eq. (24), one gets:

$$\begin{aligned} Z_2(\mathbf{Q}^0, 0) &= \frac{1}{(2\pi)^6} \int d^5\phi e^{i\mathbf{Q}^0 \cdot \phi} \exp[V \sum_j F_j(T, \gamma_s)] \\ &\times \int d\psi \exp[\sum_j \frac{(2J_j + 1)V}{(2\pi)^3} \int d^3p \log(1 + \gamma_s^{s_j} e^{-\sqrt{p^2 + m_j^2}/T - i\mathbf{q}_j \cdot \phi - i\psi})], \end{aligned} \quad (27)$$

where the first sum over j runs over all mesons and the second over all baryons. The average multiplicity of any hadron species can be derived from the global partition function (25) with the usual prescription:

$$\langle\langle n_j \rangle\rangle = \frac{\partial}{\partial \lambda_j} \log Z(\lambda_j) \Big|_{\lambda_j=1} . \quad (28)$$

The calculation of the average multiplicity of primaries according to Eq. (22) (and Eq. (28)) involves several rather complicated five-dimensional integrals which have been calculated numerically after some useful approximations, described in the following. Since the temperature is expected to be below 200 MeV, the primary production rate of all hadrons, except pions, is very well approximated by the first term of the series (22):

$$\langle\langle n_j \rangle\rangle \simeq \gamma_s^{s_j} z_j \frac{Z(\mathbf{Q}^0 - \mathbf{q}_j)}{Z(\mathbf{Q}^0)} , \quad (29)$$

where we have put $z_j \equiv z_{j(1)}$. This approximation corresponds to the Boltzmann limit of Fermi and Bose statistics. Actually, for a temperature of 170 MeV, the primary production rate of K^+ , the lightest hadron after pions, differs at most (i.e. without the strangeness suppression parameter and the chemical factors which further reduce the contribution of neglected terms) by 1.5% from that calculated with Eq. (29), well within usual experimental uncertainties. Corresponding Boltzmannian approximations can be made in the function $Z(\mathbf{Q})$, namely

$$\log(1 \pm e^{-\sqrt{p^2+m_j^2}/T - i\mathbf{q}_j \cdot \phi})^{\pm 1} \simeq e^{-\sqrt{p^2+m_j^2}/T - i\mathbf{q}_j \cdot \phi} , \quad (30)$$

which turns Eq. (20) (for a generic \mathbf{Q}) into:

$$Z(\mathbf{Q}) \simeq \frac{1}{(2\pi)^5} \int d^5\phi \, e^{i\mathbf{Q} \cdot \phi} \exp \left[\sum_j z_j \gamma_s^{s_j} e^{-i\mathbf{q}_j \cdot \phi} + \sum_{j=1}^3 \frac{V}{(2\pi)^3} \int d^3p \log(1 - e^{-\sqrt{p^2+m_j^2}/T - i\mathbf{q}_j \cdot \phi})^{-1} \right] , \quad (31)$$

where the first sum runs over all hadrons except pions and the second over pions.

As a further consequence of the expected temperature value, the z functions of all charmed and bottomed hadrons are very small: with $T = 170$ MeV and a primary production rate of K mesons of the order of one, as the data states, the z function of the lightest charmed hadron, D^0 , turns out to be $\approx 10^{-4}$; chemical factors produce a further suppression of a factor $\approx 10^{-4}$. Therefore, thermal production of heavy flavoured hadrons can be neglected, as well as their z functions in the exponentiated sum in Eq. (31), so that the integration over the variables ϕ_4 and ϕ_5 can be performed:

$$\begin{aligned} Z(\mathbf{Q}, C, B) &\simeq \frac{1}{(2\pi)^3} \int d^3\phi \, e^{i\mathbf{Q} \cdot \phi} \exp \left[\sum_j z_j \gamma_s^{s_j} e^{-i\mathbf{q}_j \cdot \phi} \right. \\ &\quad \left. - \sum_{j=1}^3 \frac{V}{(2\pi)^3} \int d^3p \log(1 - e^{-\sqrt{p^2+m_j^2}/T - i\mathbf{q}_j \cdot \phi}) \right] \delta_{C,0} \delta_{B,0} \equiv \zeta(\mathbf{Q}^0) \delta_{C,0} \delta_{B,0} . \end{aligned} \quad (32)$$

\mathbf{Q} and \mathbf{q}_j are now three-dimensional vectors consisting of electric charge, baryon number, and strangeness; the five-dimensional integrals have been reduced to three-dimensional ones.

Apart from the hadronization contribution, which is expected to be negligible in this model, production of heavy flavoured hadrons in high energy collisions mainly proceeds from hard perturbative QCD processes of $c\bar{c}$ and $b\bar{b}$ pairs creation. The fact that promptly generated heavy quarks do not reannihilate into light quarks indicates a strong deviation from statistical equilibrium of charm and beauty, much stronger than the strangeness suppression linked with γ_s .

Nevertheless, it has been found in e^+e^- collisions [5] that the relative abundances of charmed and bottomed hadrons are in agreement with those predicted by the statistical equilibrium assumption, confirming its full validity for light quarks and quantum numbers associated to them. The additional source of heavy flavoured hadrons arising from perturbative processes can be accounted for by modifying the partition function (31). In particular, the presence of one heavy flavoured hadron and one anti-flavoured hadron should be demanded in a fraction of events $R_c = \sigma(I \rightarrow c\bar{c})/\sigma(I)$ (or $R_b = \sigma(I \rightarrow b\bar{b})/\sigma(I)$) where I is the initial colliding system, namely e^+e^- , pp or $p\bar{p}$, and $\sigma(I)$ is meant to be the hadronic, inelastic and non-single-diffractive cross section respectively. Accordingly, the partition function to be used in events with a perturbative $c\bar{c}$ pair, is, by analogy with Eq. (24)-(25) and the leading baryon effect:

$$\begin{aligned} Z &= \sum_{\text{states}_1} \dots \sum_{\text{states}_N} e^{-\beta_1 \cdot P_1} \dots e^{-\beta_N \cdot P_N} \delta_{\Sigma_i \mathbf{Q}_i, \mathbf{Q}^0} \\ &- \sum_{\text{states}_1} \dots \sum_{\text{states}_N} e^{-\beta_1 \cdot P_1} \dots e^{-\beta_N \cdot P_N} \delta_{\Sigma_i \mathbf{Q}_i, \mathbf{Q}^0} \delta_{|C|,0} \equiv Z_1(\mathbf{Q}^0) - Z_2(\mathbf{Q}^0, 0), \end{aligned} \quad (33)$$

where $|C|$ is the absolute value of charm. The partition function to be used in events with a perturbative $b\bar{b}$ is, *mutatis mutandis*, analogous. Therefore, in the following analysis, we limit ourselves to the case of a $c\bar{c}$ pair, being understood that all derived results hold for the $b\bar{b}$ case as well.

In order to derive the average multiplicity of charmed hadrons in events in which one $c\bar{c}$ pair is created owing to a hard QCD process, the usual Eq. (28) should be used with the partition function Z in Eq. (33). The Z can be worked out the same way as for the leading baryon effect, namely:

$$Z = Z_1(\mathbf{Q}^0) - Z_2(\mathbf{Q}^0, 0). \quad (34)$$

where the function Z_1 can be written as in Eq. (31):

$$Z_1(\mathbf{Q}) \simeq \frac{1}{(2\pi)^5} \int d^5\phi \, e^{i\mathbf{Q}\cdot\phi} \exp \left[\sum_j z_j \gamma_s^{s_j} e^{-i\mathbf{q}_j \cdot \phi} + \sum_{j=1}^3 \frac{V}{(2\pi)^3} \int d^3p \log (1 - e^{-\sqrt{p^2+m_j^2}/T - i\mathbf{q}_j \cdot \phi})^{-1} \right], \quad (35)$$

and the function Z_2 :

$$\begin{aligned} Z_2(\mathbf{Q}^0, K) &= \frac{1}{(2\pi)^6} \int d^5\phi \int d\psi \, e^{iK\psi} \exp \left[\sum_{j=1} z_j \gamma_s^{s_j} e^{-i\mathbf{q}_j \cdot \phi - i|C_j|\psi} \right. \\ &\quad \left. + \sum_{j=1}^3 \frac{V}{(2\pi)^3} \int d^3p \log (1 - e^{-\sqrt{p^2+m_j^2}/T - i\mathbf{q}_j \cdot \phi})^{-1} \right], \end{aligned} \quad (36)$$

where the second sum in the exponentials in both Eqs. (35) and (36) runs over the three pion states and $|C_j|$ in Eq. (36) is the absolute value of j^{th} hadron's charm.

Henceforth \mathbf{Q}^0 and \mathbf{q}_i , \mathbf{q}_j , \mathbf{q}_k are to be understood as three-dimensional vectors having as components electric charge, baryon number and strangeness, while charm and beauty will be explicitly written down. By using this notation, the average multiplicity of a charmed hadron with $C_j = 1$ turns out to be (see Eq. (29)):

$$\langle\langle n_j \rangle\rangle = z_j \frac{Z_1(\mathbf{Q}^0 - \mathbf{q}_j, -1, 0) - Z_2(\mathbf{Q}^0 - \mathbf{q}_j, -1, 0, -1)}{Z_1(\mathbf{Q}^0, 0, 0) - Z_2(\mathbf{Q}^0, 0, 0, 0)} . \quad (37)$$

Since the z functions of heavy flavoured hadrons are $\ll 1$ a power expansion in the z_j 's of all charmed and anticharmed hadrons can be performed from $z_j = 0$ in the integrands of Eqs. (35) and (36), that is:

$$\exp\left[\sum_j \gamma_s^{s_j} z_j e^{-i\mathbf{q}_j \cdot \phi}\right] \simeq 1 + \sum_j \gamma_s^{s_j} z_j e^{-i\mathbf{q}_j \cdot \phi} + \frac{1}{2} \sum_{i,j} \gamma_s^{s_i} \gamma_s^{s_j} z_i z_j e^{-i(\mathbf{q}_j + \mathbf{q}_i) \cdot \phi} \quad (38)$$

for Eq. (35) and

$$\exp\left[\sum_j \gamma_s^{s_j} z_j e^{-i\mathbf{q}_j \cdot \phi - i|C_j|\psi}\right] \simeq 1 + \sum_j z_j e^{-i\mathbf{q}_j \cdot \phi - i|C_j|\psi} + \frac{1}{2} \sum_{i,j} \gamma_s^{s_i} \gamma_s^{s_j} z_i z_j e^{-i(\mathbf{q}_j + \mathbf{q}_i) \cdot \phi - 2i|C_j|\psi} \quad (39)$$

for Eq. (36). Furthermore, the z functions of the bottomed hadrons can be neglected as they are $\ll 1$ as well and beauty in Eq. (37) is always set to zero.

Those expansions permit carrying out integrations in the variables ψ , ϕ_4 and ϕ_5 in Eqs. (35) and (36). Thus:

$$Z_1(\mathbf{Q}^0 - \mathbf{q}_j, -1, 0) \simeq \sum_i \gamma_s^{s_i} z_i \zeta(\mathbf{Q}^0 - \mathbf{q}_j - \mathbf{q}_i) , \quad (40)$$

where the sum runs over the *anticharmed hadrons* as the integration in ϕ_4 of terms associated to charmed hadrons yields zero. The ζ function on the right-hand side is the same as in Eq. (32). Moreover:

$$Z_1(\mathbf{Q}^0, 0, 0) \simeq \zeta(\mathbf{Q}^0) + \sum_{i,k} \gamma_s^{s_i} \gamma_s^{s_k} z_i z_k \zeta(\mathbf{Q}^0 - \mathbf{q}_i - \mathbf{q}_k) , \quad (41)$$

where the index i runs over all charmed hadrons and index k over all anti-charmed hadrons. Owing to the presence of the absolute value of charm in the exponential $\exp[i|C_j|\psi]$ in its integrand function, the function $Z_2(\mathbf{Q}^0, C, B, K)$ vanishes if $K \leq 0$ and yields K^{th} -order terms of the power expansion in z_j if $K \geq 0$ (see Eq. (36)). Therefore:

$$Z_2(\mathbf{Q}^0, 0, 0, 0) = \zeta(\mathbf{Q}^0) \quad (42)$$

and

$$Z_2(\mathbf{Q}^0, -1, 0, -1) = 0 . \quad (43)$$

Finally, inserting Eqs. (40), (41), (42) and (43) in Eq. (37) one gets:

$$\langle\langle n_j \rangle\rangle = \gamma_s^{s_j} z_j \frac{\sum_i \gamma_s^{s_i} z_i \zeta(\mathbf{Q}^0 - \mathbf{q}_j - \mathbf{q}_i)}{\sum_{i,k} \gamma_s^{s_i} \gamma_s^{s_k} z_i z_k \zeta(\mathbf{Q}^0 - \mathbf{q}_i - \mathbf{q}_k)} , \quad (44)$$

where the indices j, k label charmed hadrons and i labels anticharmed hadrons. From previous equation it results that the overall number of charmed hadrons is 1, as it must be if c quark production from fragmentation is negligible. The average multiplicity of anticharmed hadrons is of course equal to charmed hadrons one. The same formula (44) holds for the average multiplicity of bottomed hadrons in events with a perturbatively generated $b\bar{b}$ pair. If leading

baryon effect is taken into account for $p\bar{p}$ collisions, the formula (44) gets more complicated, but the procedure is essentially the same.

It is clear that a possible charm or beauty suppression parameter γ_c or γ_b , introduced by analogy with strangeness suppression parameter γ_s , would not be revealed from the study of heavy flavoured hadron production because a single factor multiplying all z_j functions would cancel from the ratio in the right-hand side of Eq. (44).

So far, we tacitly assumed that the parameters T , V and γ_s do not fluctuate on an event by event basis. If freeze-out occurs at a fixed hadronic density in all events, then it is a reasonable *ansatz* that T and γ_s do not undergo any fluctuation since the density mainly depends on those two variables. However, there could still be significant volume fluctuations due to event by event variations of the number and size of the fireballs from which the primary hadrons emerge (see end of Sect. 2). However, it can be shown that, as far as the average hadron multiplicity ratios are concerned, possible fluctuations of V are negligible. To this end, let us define $\rho(V)$ as the probability density of picking a volume between V and $V + dV$ in a single event. The primary average multiplicity of the j^{th} hadron is then:

$$\langle\langle n_j \rangle\rangle = \int dV \rho(V) \sum_{n=1}^{\infty} (\mp 1)^{n+1} \gamma_s^{ns_j} z_{j(n)} \frac{Z(\mathbf{Q}^0 - n\mathbf{q}_j)}{Z(\mathbf{Q}^0)}. \quad (45)$$

Provided that the volume V does not fluctuate over a too large range, the chemical factors can be taken out of the integral in Eq. (45) and evaluated at the mean volume \bar{V} , because their dependence on it is mild and can be neglected (see Fig. 2). Therefore, the integrand depends on the volume only through the functions $z_{j(n)}$ whose dependence on V is linear (see Eq. (23)) and which can be reexpressed as

$$z_{j(n)}(V, T, \gamma_s) = V \xi_{j(n)}(T, \gamma_s). \quad (46)$$

Then, from Eq. (45),

$$\langle\langle n_j \rangle\rangle \simeq \sum_{n=1}^{\infty} (\mp 1)^{n+1} \gamma_s^{ns_j} \xi_{j(n)}(T, \gamma_s) \frac{Z(\mathbf{Q}^0 - n\mathbf{q}_j)}{Z(\mathbf{Q}^0)} \int dV \rho(V) V. \quad (47)$$

The integral on the right-hand side is the mean volume \bar{V} . Thus:

$$\langle\langle n_j \rangle\rangle \simeq \sum_{n=1}^{\infty} (\mp 1)^{n+1} \gamma_s^{ns_j} z_{j(n)}(\bar{V}, T, \gamma_s) \frac{Z(\mathbf{Q}^0 - n\mathbf{q}_j)}{Z(\mathbf{Q}^0)}. \quad (48)$$

It turns out that the relative hadron abundances do not depend on the volume fluctuations since the mean volume \bar{V} appearing in the above equation (replacing the volume V in Eq. (23)) is the same for all species: all results previously obtained are unaffected.

4 Fit procedure and results

The model described so far has three free parameters: the temperature T , the global volume V and the strangeness suppression parameter γ_s . They have to be determined by a fit to the available data on hadron inclusive production at each centre of mass energy. Eq. (28) yields the mean number of hadrons emerging directly from the thermal source at freeze-out, the so-called primary hadrons, [5, 12] as a function of the three free parameters. After freeze-out, primary hadrons trigger a decay chain process which must be properly taken into account

in a comparison between model predictions and experimental data, as the latter generally embodies both primary hadrons and hadrons generated by heavier particles decays. Therefore, in order to calculate overall average multiplicities to be compared with experimental data, the primary yield of each hadron species is added to the contribution stemming from the decay of heavier hadrons, which is calculated by using experimentally known decay modes and branching ratios [13, 14].

The primary average multiplicity is calculated by using Eq. (22) for pp collisions and Eq. (28) for p \bar{p} collisions. For e $^+$ e $^-$ collisions the primary production rate of the j^{th} hadron is given by:

$$\langle\langle n_j \rangle\rangle = \sum_{i=1}^5 R_i \frac{\partial}{\partial \lambda_j} \log Z_i \Big|_{\lambda_j=1} \quad (49)$$

where:

$$R_i = \frac{\sigma(e^+e^- \rightarrow q_i \bar{q}_i)}{\sigma(e^+e^- \rightarrow \text{hadrons})} \quad (50)$$

are the branching fractions into specific flavours and Z_i are the corresponding partition functions, namely Eq. (32) for u, d quarks, Eq. (33) for c quarks and also for s and b quarks where $|C|$ is replaced by $|B|$ and $|S|$ respectively. The fractions R_i depend on the centre of mass energy of the colliding system and determine the primary production of heavy flavoured hadrons to be taken into account as explained in the previous section.

As far as hadronic collisions are concerned, the fractions R_c and R_b are worst known with respect to e $^+$ e $^-$ collisions. Available data on charm cross-sections [15] indicate a fraction $R_c \approx 10^{-2} \div 10^{-3}$ at centre of mass energies < 30 GeV and, consequently, much lower values for bottom quark production. Therefore, the perturbative production of heavy quarks in hadronic collisions can be neglected as long as one deals with light flavoured hadron production at $\sqrt{s} < 30$ GeV. We assume that it may be neglected at any centre of mass energy. An upper estimate of charm production in p \bar{p} collisions at 900 GeV ($R_c = 0.3$) based on theoretical calculations [16], does not affect significantly the fitted parameters. All light flavoured hadrons and resonances with a mass < 1.7 GeV have been included among the primary generated hadron species. All heavy flavoured hadrons included in the JETSET 7.4 [14] program tables have been used.

The mass of resonances with $\Gamma > 1$ MeV has been distributed according to a relativistic Breit-Wigner function within $\pm 2\Gamma$ from the central value. The γ_s strangeness suppression factor has also been applied to neutral mesons such as ϕ , ω , etc. according to their strange valence quark content; mixing angles quoted in [13] have been used. Once the average multiplicities of the primary hadrons have been calculated as a function of the three parameters T , V and γ_s , the decay chain is performed until π , μ , K^\pm , K^0 , Λ , Ξ , Σ^\pm , Ω^- or stable particles are reached, in order to match the average multiplicity definition in pp and p \bar{p} collisions experiments. The decay chain is further extended in e $^+$ e $^-$ collisions since experiments also include the decay products of K_s^0 , Λ , Ξ , Σ^\pm and Ω^- in their multiplicity definition. Finally, the overall yield is compared with experimental measurements, and the χ^2 :

$$\chi^2 = \sum_i (\text{theo}_i - \text{expe}_i)^2 / \text{error}_i^2 \quad (51)$$

is minimized.

As far as the data set is concerned, we used all available measurements of hadron multiplicities in non-single-diffractive p \bar{p} [17] and inelastic pp collisions [18] down to a centre of mass energy

of about 19 GeV and e^+e^- collisions between 29 and 91.2 GeV [19].

Whenever several measurements at the same centre of mass energy have been available, averages have been calculated according to a weighting procedure described in ref. [20] prescribing rescaling of errors to take into account *a posteriori* correlations and disagreements of experimental results. If an experiment repeated the same measurement at the same centre of mass energy, we used only the most recent value.

Since the decay chain is an essential step of the fitting procedure, calculated theoretical multiplicities are affected by experimental uncertainties on masses, widths and branching ratios of all involved hadron species. In order to estimate the effect of these uncertainties on the results of the fit, a two-step procedure for the fit itself has been adopted: firstly, the fit has been performed with a χ^2 including only experimental errors and a set of parameters T_0 , V_0 , γ_{s0} has been obtained. Then, the various masses, widths and branching ratios have been varied in turn by their errors, as quoted in ref. [13], and new theoretical multiplicities calculated, keeping the parameters T_0 , V_0 , γ_{s0} fixed. The differences between old and new theoretical multiplicity values have been considered as additional systematic errors to be added in quadrature to experimental errors. Finally, the fit has been repeated with a χ^2 including overall errors so as to obtain final values for model parameters and for theoretical multiplicities. Among the mass, width and branching ratio uncertainties, only those producing significant variations of final hadron yields (actually more than 130) have been considered.

The fitted values of the parameters T , V , γ_s at various centre of mass energy points are quoted in Table 1. The fit quality is very good at almost all centre of mass energies as demonstrated by the low values of χ^2 's and by the Figs. 3, 4, 5, 6, 7. Owing to the relatively large value of χ^2 at $\sqrt{s} = 27.4$ GeV in pp collisions and at $\sqrt{s} = 91.2$ GeV in e^+e^- collisions, variations of fitted parameters larger than fit errors must be expected when repeating the fit excluding data points with the largest deviations from the theoretical values. Therefore, the fit at $\sqrt{s} = 27.4$ GeV pp collisions has been repeated excluding in turn (Δ^0, ρ^0, ϕ) and (K^-, pions) , respectively, from the data set; the fit at $\sqrt{s} = 91.2$ GeV e^+e^- collisions has been repeated excluding $\Sigma^{*\pm}$. The maximum difference between the new and old fit parameters has been considered as an additional systematic error and is quoted in Table 1 within brackets. The fitted temperatures are compatible with a constant value at freeze-out independently of collision energy and kind of reaction (see Fig. 8) and it is in good agreement with that found in heavy ions collisions [21]. The γ_s parameter exhibits a very slow rise from 20 to 900 GeV (see Fig. 9); its value of $\simeq 0.5$ over the whole explored centre of mass energy range in hadronic collisions proves that complete strangeness equilibrium is not attained. Furthermore, γ_s appears to be definitely lower in pp and p \bar{p} collisions than in e^+e^- collisions at equal centre of mass energy.

On the other hand, the global volume does increase as a function of centre of mass energy as it is proportional, for nearly constant T and γ_s , to overall multiplicity which indeed increases with energy. Its values range from 6.4 fm³ at $\sqrt{s} = 19.4$ GeV pp collisions, at a temperature of 191 MeV, up to 67 fm³ at $\sqrt{s} = 900$ GeV p \bar{p} collisions at a temperature of 170 MeV.

Once T , V and γ_s are determined by fitting average multiplicities of some hadron species, their values can be used to predict average multiplicities of any other species, at a given centre of mass energy.

The physical significance of the results found so far depends on their stability as a function of the various approximations and assumptions which have been introduced. First, the temperature and γ_s values are low enough to justify the use of the Boltzmann limits (29), (30) for all hadrons except pions, as explained in Sect. 3. As far as the effect of a cut-off in the hadronic mass spectrum goes, the most relevant test proving that our results do not depend

Table 1: Values of fitted parameters in pp, p \bar{p} and e $^+$ e $^-$ collisions.

\sqrt{s} (GeV)	T (MeV)	VT^3	γ_s	χ^2/dof
pp collisions				
19.5	190.8 ± 27.4	5.8 ± 3.1	0.463 ± 0.037	6.38/4
23.8	194.4 ± 17.3	6.3 ± 2.5	0.460 ± 0.067	2.43/2
26.0	159.0 ± 9.5	13.4 ± 2.7	0.570 ± 0.030	1.86/2
27.5	169.0 ± 2.1 (± 3.4)	11.04 ± 0.69 (± 1.4)	0.510 ± 0.011 (± 0.025)	136.4/27
p \bar{p} collisions				
200	175.4 ± 14.8	24.3 ± 7.9	0.537 ± 0.066	0.698/2
546	181.7 ± 17.7	28.5 ± 10.4	0.557 ± 0.052	3.80/1
900	170.2 ± 11.8	43.2 ± 11.8	0.578 ± 0.063	1.79/2
e $^+$ e $^-$ collisions				
29 \div 30	163.6 ± 3.6	15.2 ± 1.4	0.724 ± 0.045	24.7/13
34 \div 35	165.2 ± 4.4	14.6 ± 1.6	0.788 ± 0.045	10.5/8
42.6 \div 44	169.6 ± 9.5	14.7 ± 3.1	0.730 ± 0.060	4.94/4
91.2	160.6 ± 1.7 (± 3.1)	26.4 ± 1.3 (± 1.1)	0.675 ± 0.020 (± 0.027)	60.8/21

on it is the stability of the number of primary hadrons against changes of the cut-off mass. The fit procedure intrinsically attempts to reproduce fixed experimental multiplicities; if the number of primary hadrons does not change significantly by repeating the fit with a slightly lower cut-off, the production of heavier hadrons excluded by the cut-off must be negligible, in particular with regard to its decay contributions to light hadron yields. In this spirit all fits have been repeated moving the mass cut-off value from 1.7 down to 1.3 GeV in steps of 0.1 GeV, checking the stability of the amount of primary hadrons as well as of the fit parameters. It is worth remarking that the number of hadronic states with a mass between 1.7 and 1.6 GeV is 238 out of 535 overall, so that their exclusion is really a severe test for the reliability of the final results. Figure 10 shows the model parameters and the primary hadrons in p \bar{p} collisions at $\sqrt{s} = 900$ GeV; above a cut-off of 1.5 GeV the number of primary hadrons settles at an asymptotically stable value, whilst the fitted values for T , V , γ_s do not show any particular dependence on the cut-off. Therefore, we conclude that the chosen value of 1.7 GeV ensures that the obtained results are meaningful.

5 Conclusions

A thermodynamical approach to hadronization, based on the use of the canonical formalism of statistical mechanics ensuring the exact quantum numbers conservation, succeeds in reproducing the average multiplicities of hadron species in e $^+$ e $^-$, pp and p \bar{p} collisions. These quantities are best suited in investigating the main characteristics of hadronization because of their independence from local collective flows. The success of this statistical approach indicates that,

apart from the violation of strangeness phase space saturation as implemented in the γ_s parameter, hadron production in elementary high energy collisions is dominated by phase space rather than by dynamical effects. The dynamics is essentially contained in hadron gas fireballs collective motions reflecting local hard partons kinematics.

The temperature at freeze-out seems to be constant independently of kind of reaction and centre of mass energy. This uniformity suggests that parton-hadron transition cannot occur before the parameters of prehadronic matter such as density or pressure have dropped below critical values corresponding to a temperature of about 170 MeV in a (partially) equilibrated hadron gas. This value is in agreement with lattice QCD results [22].

From previous results it turns out that strangeness suppression, as well as the survival of perturbatively created $c\bar{c}$ and $b\bar{b}$ pairs are the only trace to strong interaction dynamics before hadronization. This finding suggests that such non-equilibrium effects are mainly related to quark mass thresholds.

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π^+ : TASSO Coll., *Z. Phys. C* **42**, 189 (1989)

π^0 : TASSO Coll., *Z. Phys. C* **33**, 13 (1985); CELLO Coll., *Z. Phys. C* **47**, 1 (1990); JADE Coll., *Z. Phys. C* **46**, 1 (1990)

η : CELLO Coll., *Z. Phys. C* **47**, 1 (1990); JADE Coll., *Z. Phys. C* **46**, 1 (1990)

K^+ : TASSO Coll., *Z. Phys. C* **42**, 189 (1989)

K^0 : TASSO Coll., *Z. Phys. C* **27**, 27 (1985); TASSO Coll., *Z. Phys. C* **47**, 167 (1990); JADE Coll., *Z. Phys. C* **20**, 187 (1983); CELLO Coll., *Z. Phys. C* **46**, 397 (1990)

K^{*+} : CELLO Coll., *Z. Phys. C* **46**, 397 (1990); JADE Coll., *Phys. Lett. B* **145**, 441 (1984); TASSO Coll., *Z. Phys. C* **47**, 167 (1990)

p: JADE Coll., *Phys. Lett. B* **104**, 325 (1981); TASSO Coll., *Z. Phys. C* **42**, 189 (1989)

Λ : TASSO Coll., *Z. Phys. C* **27**, 27 (1985); TASSO Coll., *Z. Phys. C* **45**, 209 (1989); CELLO Coll., *Z. Phys. C* **46**, 397 (1990)

Ξ^- : TASSO Coll., *Z. Phys. C* **27**, 27 (1985); TASSO Coll., *Z. Phys. C* **45**, 209 (1989);

For $\sqrt{s} = 42.6 \div 44$ GeV collisions:

Charged particles: TASSO Coll., *Z. Phys. C* **45**, 193 (1989)

π^+ : TASSO Coll., *Z. Phys. C* **42**, 189 (1989)

π^0 : JADE Coll., *Z. Phys. C* **46**, 1 (1990)

K^+ : TASSO Coll., *Z. Phys. C* **45**, 193 (1989)

K^0 : TASSO Coll., *Z. Phys. C* **47**, 167 (1990)

K^{*+} : TASSO Coll., *Z. Phys. C* **47**, 167 (1990)

Λ : TASSO Coll., *Z. Phys. C* **45**, 209 (1989)

For $\sqrt{s} = 91.2$ GeV collisions:

π^+ : OPAL Coll., *Z. Phys. C* **63**, 181 (199)

π^0 : L3 Coll., *Phys. Lett. B* **328**, 223 (1994); DELPHI Coll., *Z. Phys. C* **69**, 561 (1996); ALEPH Coll., CERN/PPE 96-168 (1996)

η : ALEPH Coll., *Phys. Lett. B* **292**, 210 (1992), extrapolated by P. Chliapnikov, V. A. Uvarov, *Phys. Lett. B* **345**, 313 (1995); L3 Coll., *Phys. Lett. B* **328**, 223 (1994)

ρ^0 : ALEPH Coll., *Z. Phys. C* **69**, 379 (1996); DELPHI Coll., *Z. Phys. C* **65**, 587 (1994)

ω : ALEPH Coll., *Z. Phys. C* **69**, 379 (1996)

η' : ALEPH Coll., *Phys. Lett. B* **292**, 210 (1992), extrapolated by P. Chliapnikov, V. A. Uvarov, *Phys. Lett. B* **345**, 313 (1995)

ϕ : ALEPH Coll., *Z. Phys. C* **69**, 379 (1996); OPAL Coll., *Z. Phys. C* **68**, 1 (1995); DELPHI Coll., CERN/PPE 96-77 (1996)

$f_2'(1525)$: DELPHI Coll. *Phys. Lett. B* **379**, 309 (1996); K^+ : DELPHI Coll., *Nucl. Phys. B* **444**, 3 (1995); OPAL Coll., *Z. Phys. C* **63**, 181 (1994)

K^0 : ALEPH Coll., *Z. Phys. C* **64**, 361 (1994); DELPHI Coll., *Z. Phys. C* **65**, 587 (1995); L3 Coll., *Phys. Lett. B* **328**, 223 (1994); OPAL Coll., *Z. Phys. C* **67**, 389 (1995)

K^{*+} : DELPHI Coll., *Z. Phys. C* **65**, 587 (1995); OPAL Coll., *Phys. Lett. B* **305**, 407 (1993)

K^{*0} : ALEPH Coll., *Z. Phys. C* **69**, 379 (1996); OPAL Coll., *Z. Phys. C* **68**, 1 (1995); DELPHI Coll., CERN/PPE 96-77 (1996)

$K_2^{*0}(1430)$: DELPHI Coll., CERN/PPE 96-77 (1996)

p : DELPHI Coll., *Nucl. Phys. B* **444**, 3 (1995); OPAL Coll., *Z. Phys. C* **63**, 181 (1994)
 Δ^{++} : DELPHI Coll., *Phys. Lett. B* **361**, 207 (1995); OPAL Coll., *Phys. Lett. B* **358**, 162 (1995)
 Λ : DELPHI Coll., *Phys. Lett. B* **318**, 249 (1993); L3 Coll., *Phys. Lett. B* **328**, 223 (1994); ALEPH Coll., *Z. Phys. C* **64**, 361 (1994); OPAL Coll., CERN/PPE 96-099 (1996)
 $\Sigma^{*\pm}$: ALEPH Coll., contributed paper EPS0419 to the 1995 European Physical Society conference (Brussels); DELPHI Coll., *Z. Phys. C* **67**, 543 (1995); OPAL Coll., CERN/PPE 96-099 (1996)
 Σ^- : DELPHI Coll., contributed paper pa01-40 to the 1996 I.C.H.E.P. conference (Warsaw); OPAL Coll., CERN/PPE 96-099 (1996)
 Σ^+ : OPAL Coll., CERN/PPE 96-099 (1996)
 Σ^0 : DELPHI Coll., *Z. Phys. C* **70**, 371 (1996); OPAL Coll., CERN/PPE 96-099 (1996)
 Ξ^- : DELPHI Coll., contributed paper pa01-109 to the 1996 I.C.H.E.P. conference (Warsaw); ALEPH Coll., contributed paper EPS0419 to the 1995 European Physical Society conference (Brussels); OPAL Coll., CERN/PPE 96-099 (1996)
 Ξ^{*0} : DELPHI Coll., *Z. Phys. C* **67**, 543 (1995); ALEPH Coll., contributed paper EPS0419 to the 1995 European Physical Society conference (Brussels); OPAL Coll., CERN/PPE 96-099 (1996)
 Ω : DELPHI Coll., *Z. Phys. C* **70**, 371 (1996); ALEPH Coll., contributed paper EPS0419 to the 1995 European Physical Society conference (Brussels); OPAL Coll., CERN/PPE 96-099 (1996)
 $\Lambda(1520)$: OPAL Coll., CERN/PPE 96-099 (1996).

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P. Braun-Munzinger et al., *Phys. Lett. B* **365** 1 (1996)
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Figure captions

- Figure 1 Behaviour of the global partition function Z as a function of electric charge, baryon number and strangeness, keeping all remaining quantum numbers set to zero, for $T = 170$ MeV, $V = 20 \text{ fm}^3$ and $\gamma_s = 0.5$
- Figure 2 Behaviour of the non-strange baryon chemical factor $Z(0, 1, 0, 0, 0)/Z(0, 0, 0, 0, 0)$ as a function of volume for different values of the temperature and a fixed value $\gamma_s = 0.5$ for the strangeness suppression parameter
- Figure 3 Results of hadron multiplicity fits for pp collisions at centre of mass energies 19.4, 23.8 and 26 GeV. Experimental average multiplicities are plotted versus calculated ones. The dashed lines are the quadrant bisectors: well fitted points tend to lie on these lines
- Figure 4 Results of hadron multiplicity fit for pp collisions at centre of mass energy of 27.4 GeV. Top: the experimental average multiplicities are plotted versus the calculated ones. The dashed line is the quadrant bisector; well fitted points tend to lie on this line. Bottom: residual distributions
- Figure 5 Results of hadron multiplicity fit for $p\bar{p}$ collisions at centre of mass energies 200, 546 and 900 GeV. Experimental average multiplicities are plotted versus calculated ones. The dashed lines are the quadrant bisectors: well fitted points tend to lie on these lines
- Figure 6 Results of hadron multiplicity fits for e^+e^- collisions at centre of mass energies 29, 35 and 44 GeV. Experimental average multiplicities are plotted versus calculated ones. The dashed lines are the quadrant bisectors: well fitted points tend to lie on these lines
- Figure 7 Results of hadron multiplicity fits for e^+e^- collisions at centre of mass energy of 91.2 GeV. Experimental average multiplicities are plotted versus calculated ones. The dashed lines are the quadrant bisectors: well fitted points tend to lie on these lines
- Figure 8 Freeze-out temperature values as a function of centre of mass energy. The error bars within horizontal ticks for 27.4 GeV pp collisions and for 91.2 GeV e^+e^- collisions are the fit errors; the overall error bars are the sum in quadrature of the fit error and the systematic error related to data set variation (see text)
- Figure 9 Strangeness suppression parameters γ_s as a function of centre of mass energy. The error bars within horizontal ticks for 27.4 GeV pp collisions and for 91.2 GeV e^+e^- collisions are the fit errors; the overall error bars are the sum in quadrature of the fit error and the systematic error related to data set variation (see text)
- Figure 10 Dependence of fitted parameters and primary average multiplicities (top) on the mass cut-off in the hadron mass spectrum for $p\bar{p}$ collisions at a centre of mass energy of 900 GeV

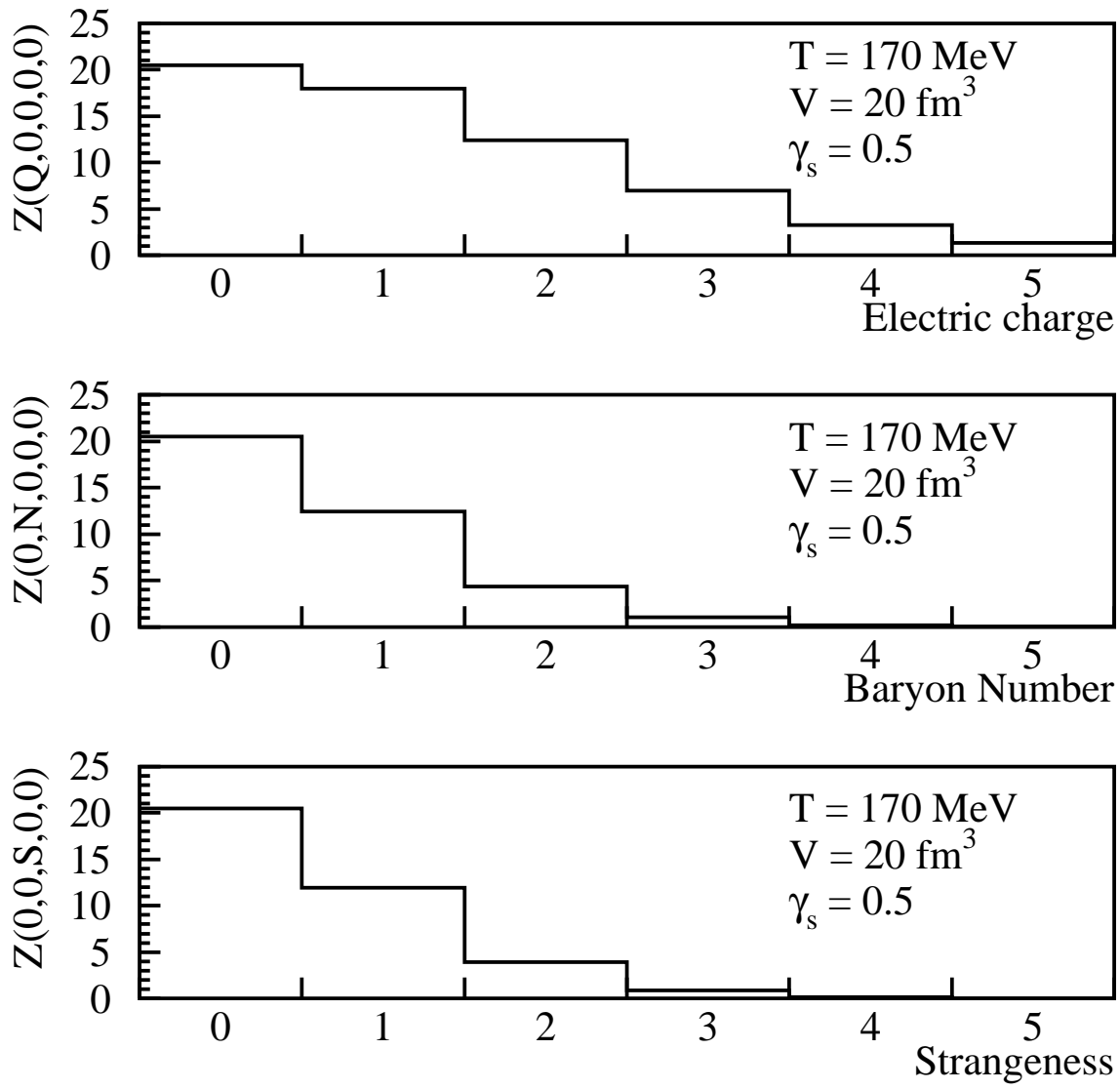


Figure 1:

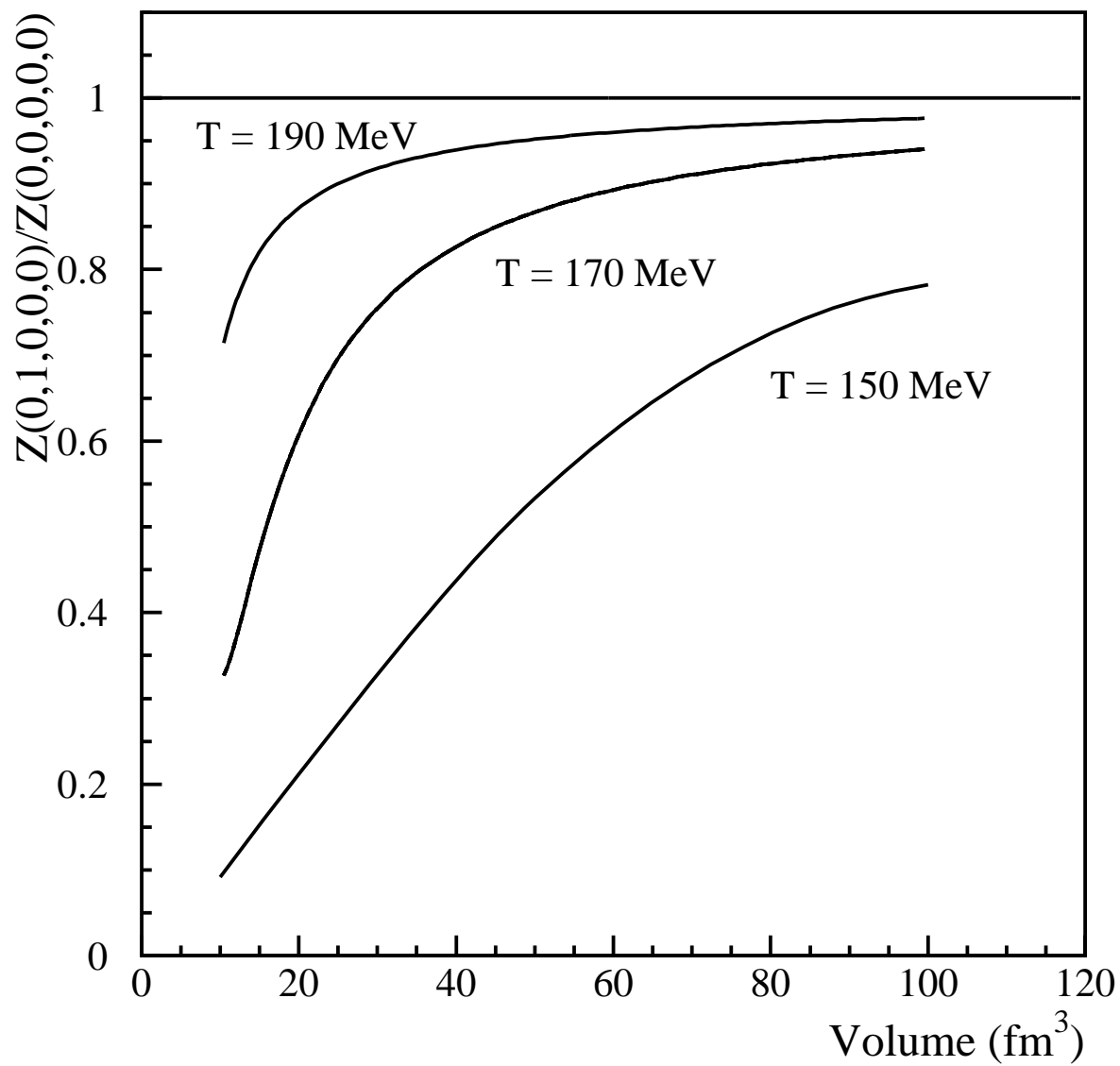


Figure 2:

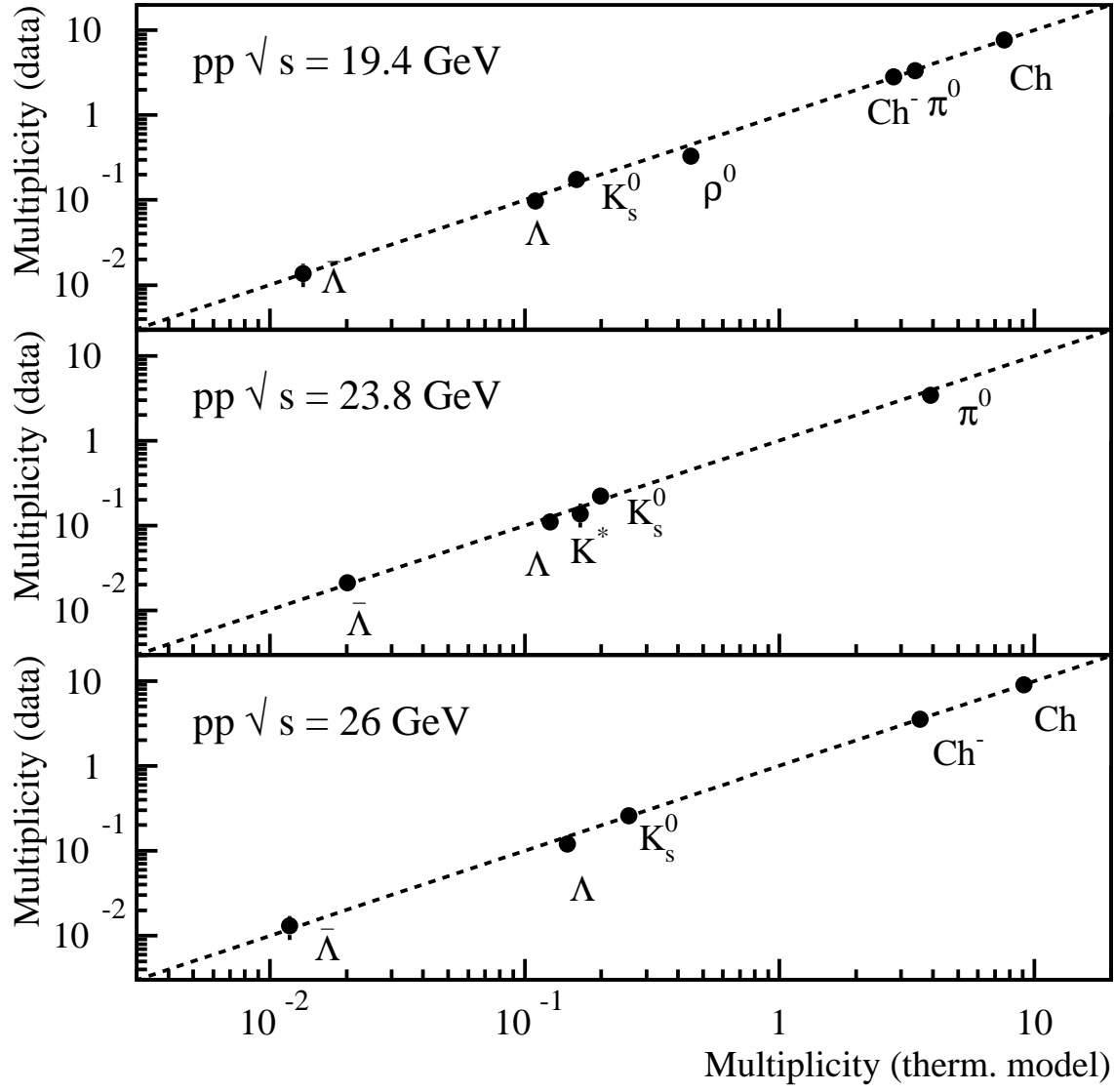


Figure 3:

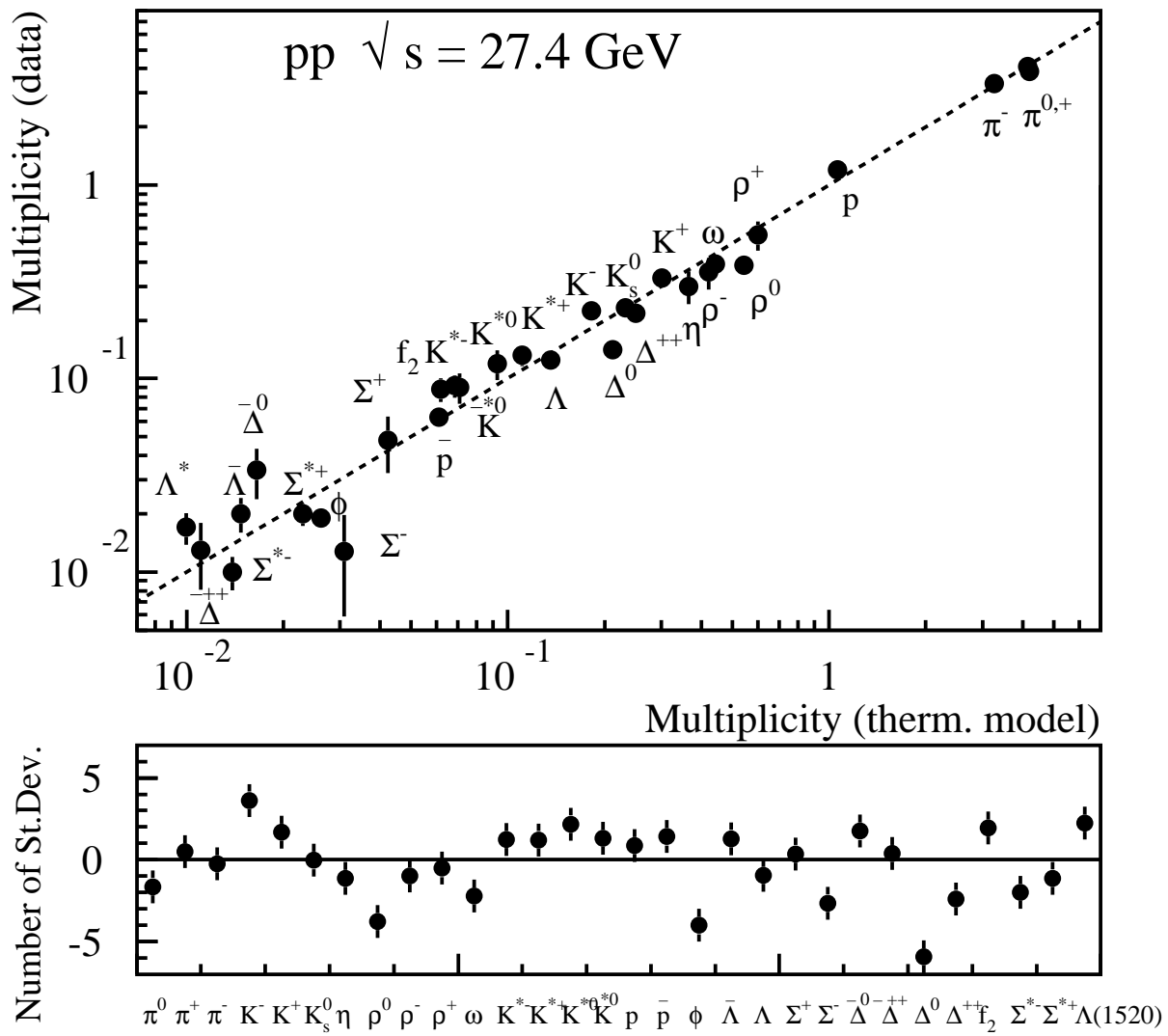


Figure 4:

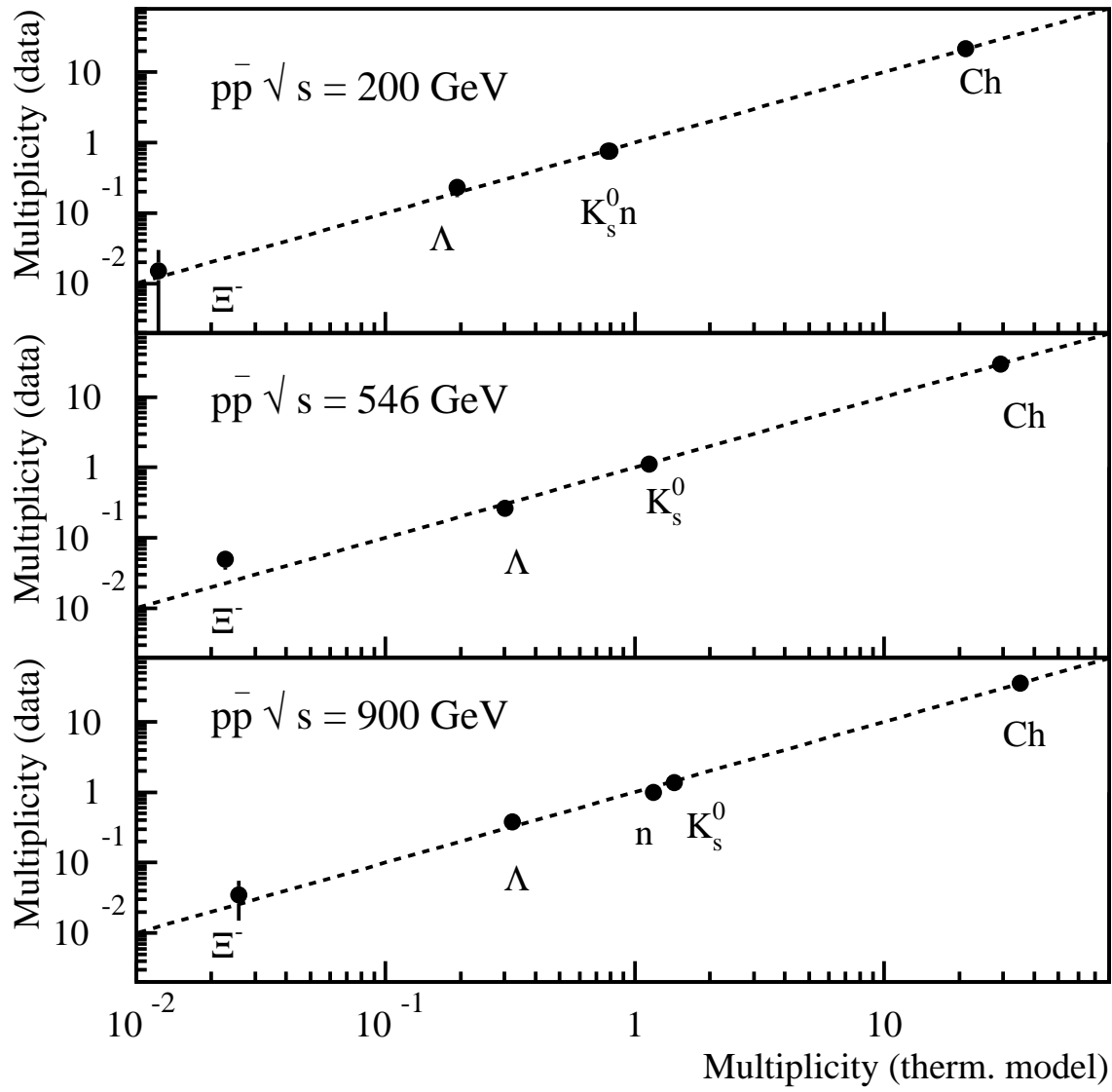


Figure 5:

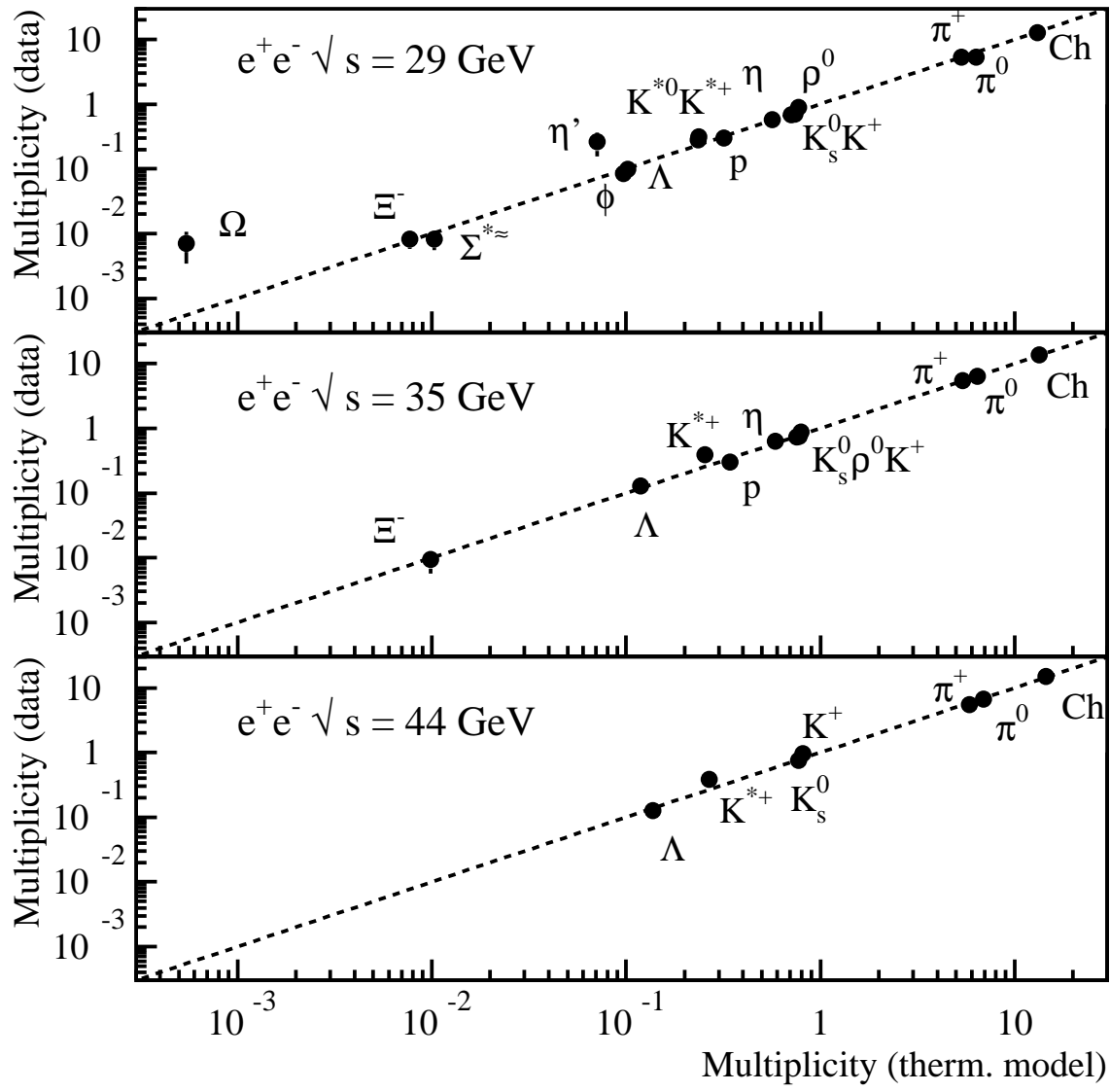


Figure 6:

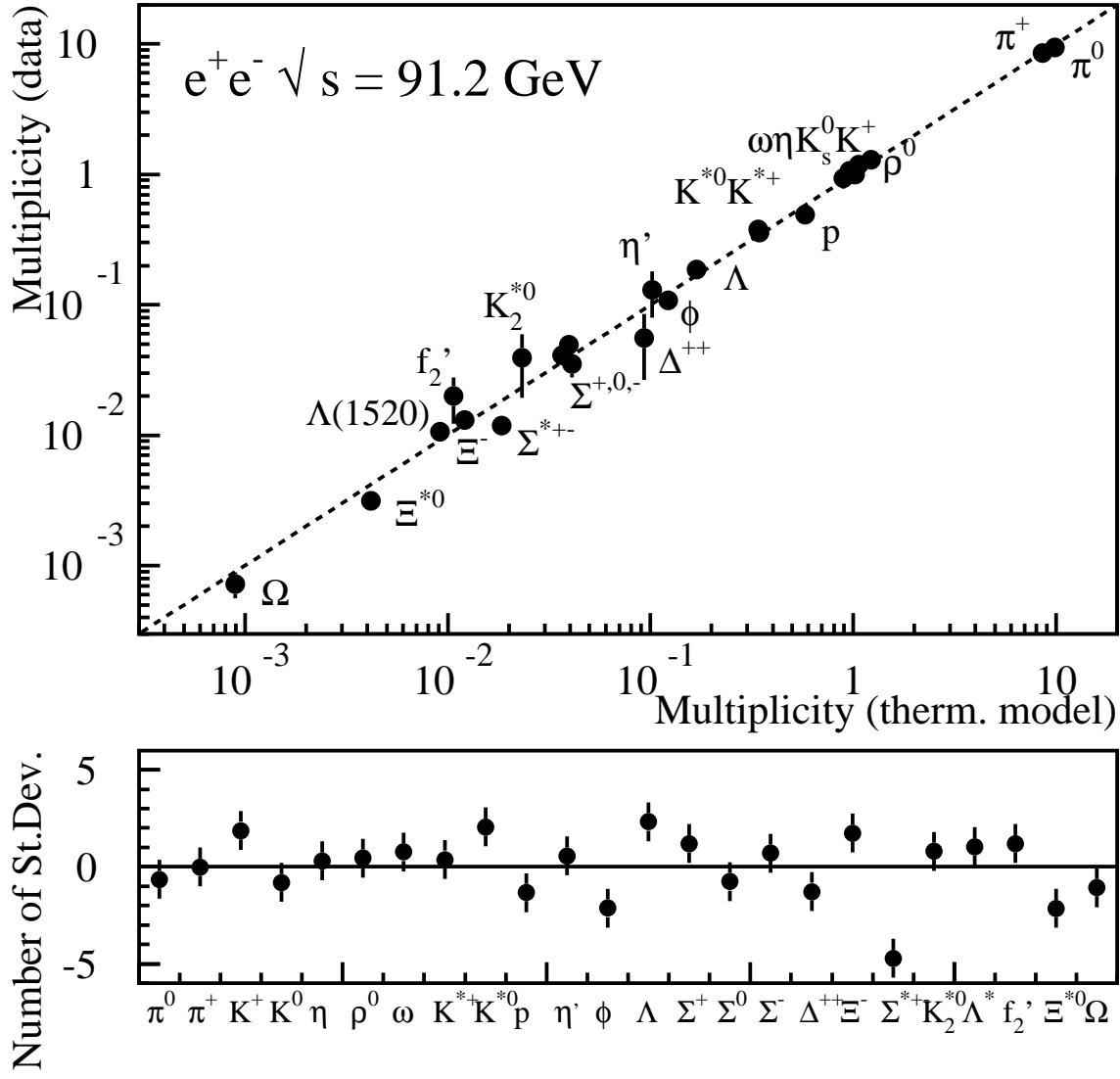


Figure 7:

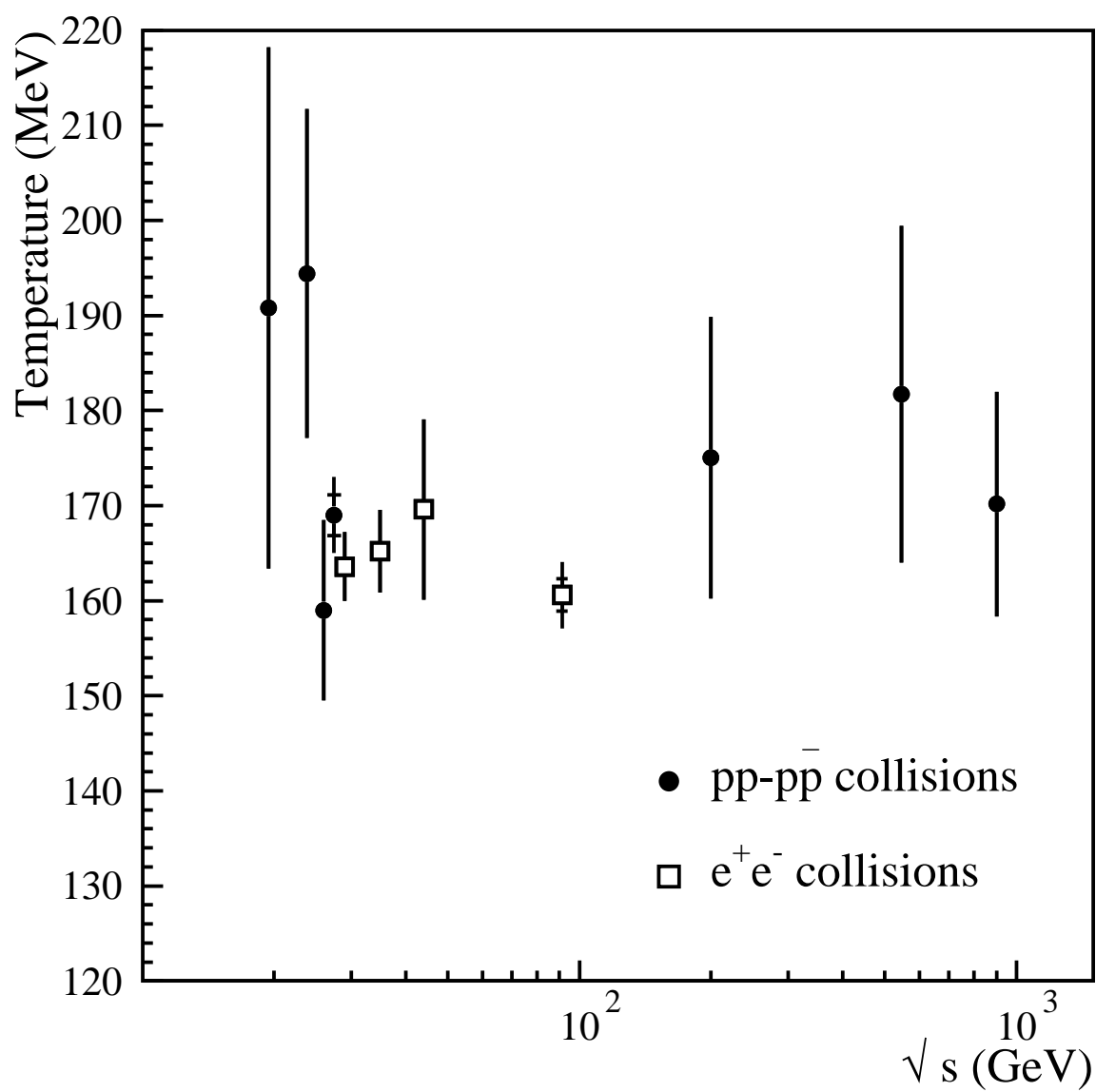


Figure 8:

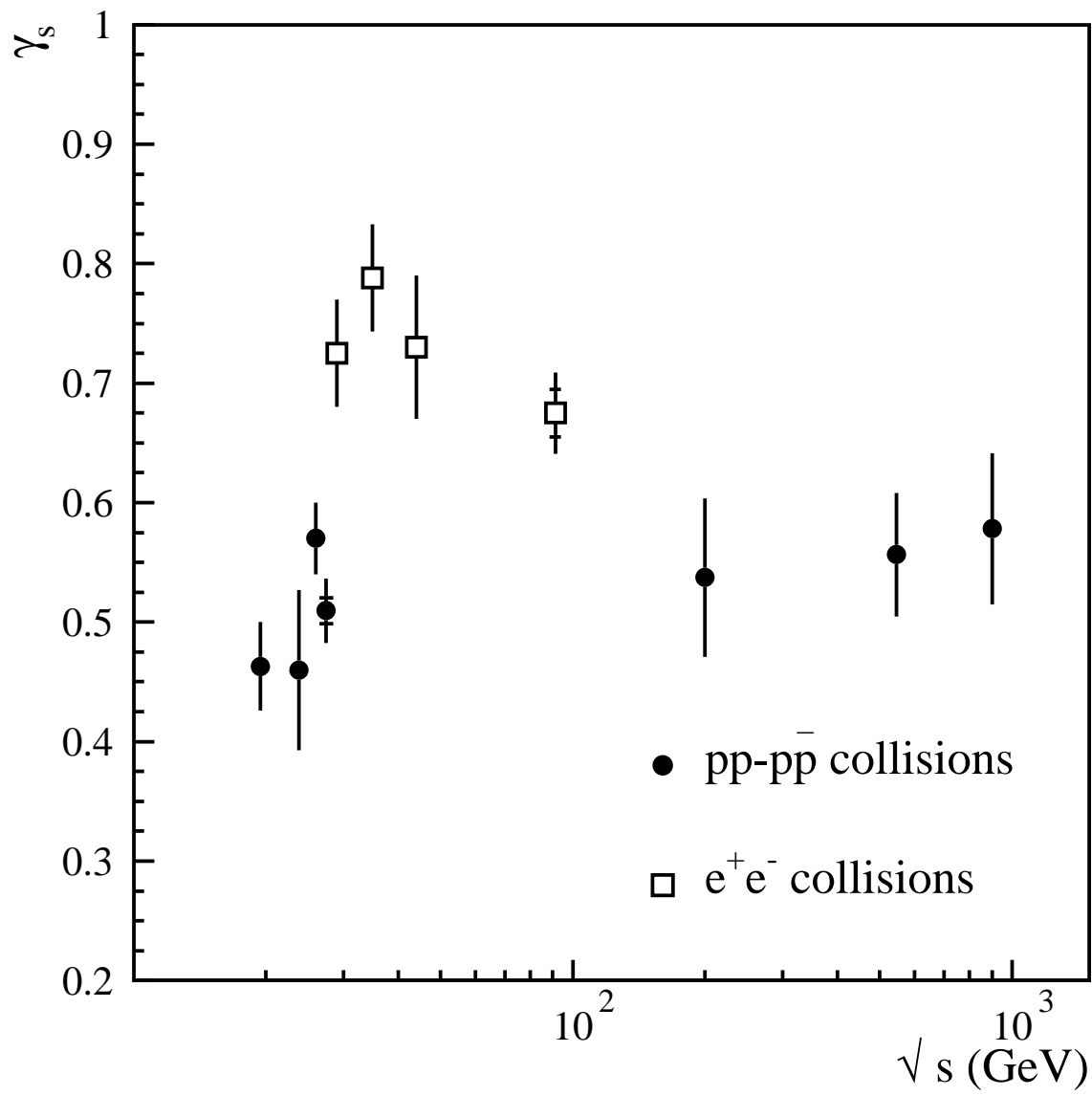


Figure 9:

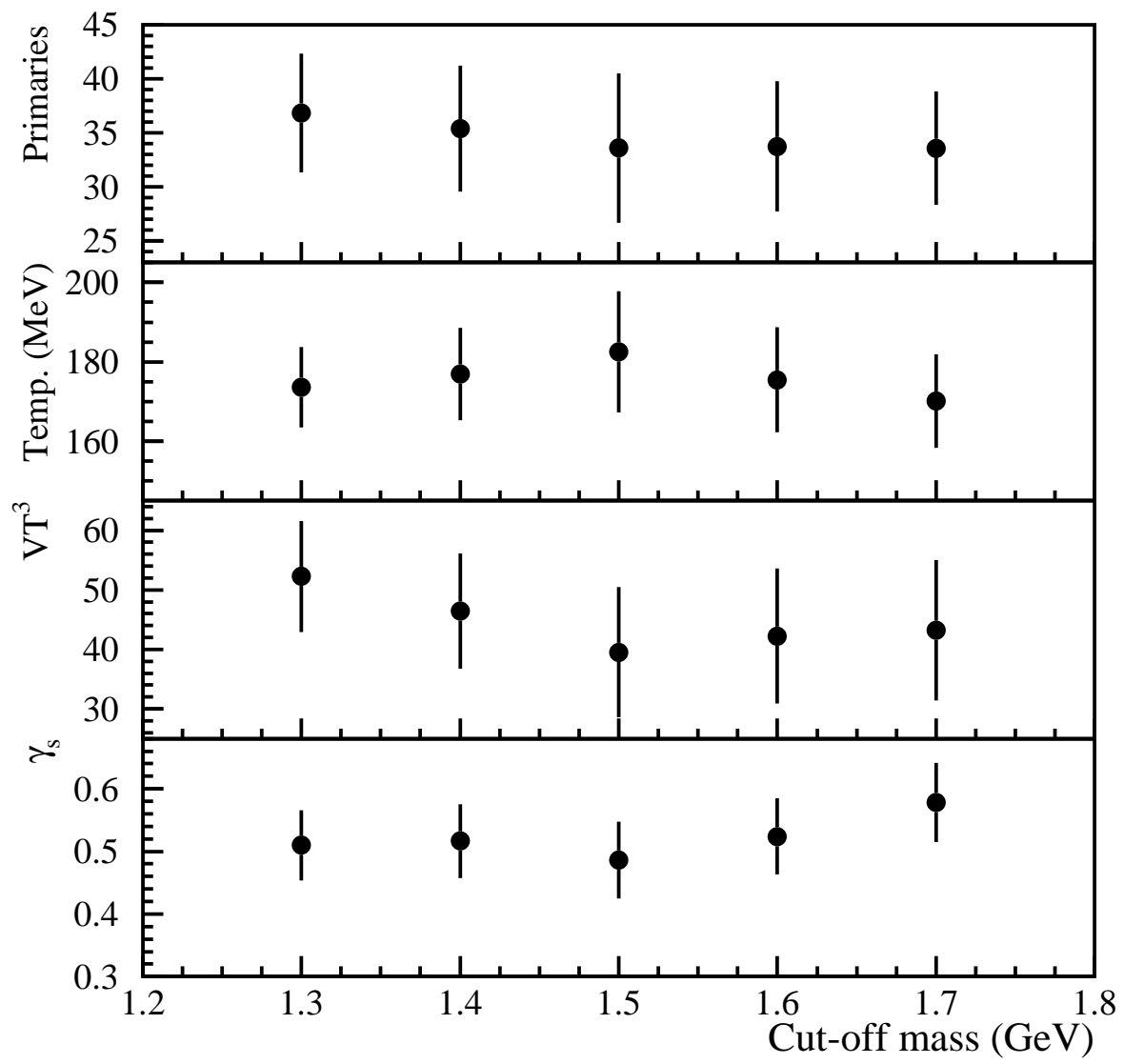


Figure 10: